QUANTIFICATION IN AFRICAN LOGIC

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1. Predication

By predication alone, Africans say many things with seeming ease which ordinarily they would not and could not say. The metalogical beauty of it is that they say without having said and they make hills flat without having lifted a hoe. In this one finds African predicate logic a lot richer than its western counterpart. Predicate logic, sometimes called quantification logic was invented by the German Logician Gottlob Frege[1] in his monumental book *Bergriffsschrift*. It has since been broken down to a number of classifications namely first-order, second-order and higher-order. In African demarcation, we shall treat just the first and the second order. The mainline of difference between the western and the African versions of these logics are to be found in the quantifiers, rules, evaluations, operators, variables, proof mechanisms and the criterion for logically valid formulae. For the latter, while validity depends on subject matter in African logic, in western logic it depends primarily on logical form. Logical form in its secondary role is just like another tool in a kit box for African logic. In what follows, I shall outline the main doctrines of the first and the second order logics.

[1] Bergriffcshrift". *From Frege to Godel: A Source Book in Mathematical Logic, 1879-1931*. Ed. Heijenoort, van
Jean. Cambrigde: Harvard University Press, 1967. Print.

African first-order logic lays additional blocks on top of its propositional logic earlier discussed. Some of such main additions include:

- Statements that ascribe a predicate to an individual e.g. Okonkwo[2] is brave, we symbolize this as Bo. Notice that the predicate constant is written in upper case and appears before the subject constant. This is because in this logic, attention shifts from the subject (as in propositional logic) to the predicate (what is being said of the subject).

This shift accounts in part for the massive expressive power of this logic and of course for this focus on predicates, it is called predicate logic sometimes.

- Statements that ascribe a relation to individuals, e.g. Ihuoma[3] was a concubine of Emenike, we symbolize this as Cie.
- Quantified statements which, say that a certain predicate or relation applies to some individuals e.g. at least some persons are brave, we symbolize this as (($GH_{Ọ}$)Bọ). Here we employ the upper case of the Igbo twin alphabet GH as existential quantifier (some) and one of the Igbo dotted letters ọ as a variable.
- Quantified statements which, say that a certain predicate or relation applies to one individual e.g. one person is brave, we symbolize this as (($GB_{Ọ}$)Bọ). Here we employ the uppercase of the Igbo twin alphabet GB as existential quantifier (one). Notice therefore that unlike in western logic, African logic does not issue the same quantification to the expressions "one" and "some". The expression "at least" covers "some" but it is unnecessary when the object is only "one". Thus for clarity of thought in African logic we quantify some and one differently.

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[2] Okonkwo, Unoka and Ezeudo are some of the characters in Achebe, Chinua. *Things Fall Apart*. London: Heinemann, 1958

[3] Ihuoma and Emenike are some of the characters in Amadi, Elechi. *The Concubine*. London: Heinemann.

- Quantified statements which, say that a certain predicate or relation applies to every individual e.g. every African is strong, we symbolize this as (KWọ) (Aọ $_{M}$Sọ). Notice that we intuitively assigned a context indicator M because the subject matter reveals that what is said of the African occurs in the para-contingent world that is. $\mapsto$
- Multi-quantified statements in which, the variables stand for individuals e.g., everything is caused by something, we can symbolize this as (KWọ) [(GHụ) (Cọụ)]. Notice also that everyman is created by one God attracts existential quantifier (one) i.e. (KWọ) (GBụ) (Cọụ). One point to remember is that all the statements of first-order logic are about individual entities. The second order logic varies in that it focuses mainly on predicates and relations. Thus, African second-order logic like its western counterpart adds to first-order logic, the logic of statements concerning predicates and relations e.g. there is a predicate that applies both to Unoka and Okonkwọ, we may symbolize this as (GBP) (Pụ $\wedge$ Pọ); notice that we employ the uppercase letter P as both the quantified constant and the predicate constant. The reason for using it as an upper case quantified constant is to distinguish it from the individual variable. On relations, we take the example; “there is a property that belongs to everything”, we may symbolize this as; (GBP) (KWọ) Pọ. Notice again that we employ upper case letter P as a quantified constant for property or relation and as predicate constant. On the whole, the student of African logic should ultimately focus on what is being quantified in second order logic. It is either a predicate or a relation constant and not an individual variable as in first-order logic. Also, the two examples above could well be rewritten “there are some predicates that apply both to Unoka and Okonkwọ” and “there are some properties that belong to everything”. This changes the existential quantifier from one (GB) to some (GH) and by so doing further increases the

expressive power of African second order logic. The above two statements may now be symbolized as follows.

(GHP) (Pụ $\wedge$ Pọ) and

(GHP) (KWọ) Pọ

With this at hand, let us now deal with the syntax and semantics of African predicate logic.

2. Syntax

Every logical system has both the syntactic and the semantic components. The function of syntax is to determine which, array of symbols are legal expressions within the system while that of semantics is to determine the meanings behind these expressions. African predicate logic has a language and a set of alphabets different from ordinary language like Akan, Igbo, Zulu, Swahili etc., this language is formal but unlike the western logic, it is not completely formal. A logical language is formal when it is constituted of strings of symbols which obey the rule of consequence relation such that it can be mechanically determined whether a given expression is legal or a formula valid. But the language of African predicate logic is said to be customary[4] rather than completely formal thus in testing the legality of expression or the validity of formulae, the African logician goes beyond logical form and appeals ultimately to logical custom and this is also done mechanically.

In all standard logics, there are two main types of legal expressions: terms, which intuitively represent objects and

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[4] Okeke Chimakonam, J. "Why Can't There be an African logic?". *Journal of Integrative Humanism*. 1.
2. (2011). 141-152. Print. P.148. Other scholars like Udo Etuk. The Possibility of Igbo-African logic". *The Third Way in African Philosophy*, Olusegun Oladipo (ed). Ibadan: Hope Publications, 2002. Print." would prefer the term 'affective'. This latter term is similar to Lepold Senghor's much misinterpreted and misunderstood term 'emotion' in his Liberte I: Negritude et Humanisme. Paris: Editions du Seuil, 1964. Pp23-24

formulae, which intuitively express predicates that can be true or false. The terms and formulae of predicate logic are strings of symbols which together form the alphabet of the language. In a customary language therefore, the nature of the symbols themselves is outside the scope of formal logic because they are not merely place holders that maintain formal order but are supposed to reflect realities around. They also function simply as letters and punctuation symbols.

Let us also divide the symbols of the alphabet into logical symbols, which, always have the same meaning, and non-logical symbols, whose meanings vary by interpretation. The wedged-implication sign |⟶ always represents the expression "if then through …" and is never interpreted as "and"∧. But a non-logical predicate symbol such as schol (ọ) could be interpreted to, mean "ọ is a scholar), "ọ is a teacher", "ọ is a mouse" or just any expression with a unary predicate.
The basic logical symbols of African predicate logic include: quantifier symbols KW (Universal), GH (existential some) and GB (existential one). The logical connective, ∧ for na or conjunction (and); ∨ for ma- ọbụ disjunction (or); |⟶ for site-na wedged-implication ⟵| ; for sitelu-na wedged-reduction, ⇔ for bụ-otu equivalence, ~ for ọbụghị̣ negation. Punctuations, brackets, parenthesis, braces, commas, diagrams and others as the context may demand. Variables, an infinite strings of lower case letter starting with the Igbo dotted letters ọ ụ ị, i, … usually used in denoting arbitrary individuals. Constants, an infinite string of most times upper case letters, usually the first of the predicate term or the individual as the case may be. Subscripts, strings of lower case letters or numbers used in distinguishing variables usually lowered down in front of the variables e.g. ọ$_{ø}$, ọ ⌉ ọ ⌋, … . Superscripts, strings of lower case letters or numbers used in distinguishing variables usually higher up in front of the variables e.g. ụ^{n}, ụ^{m}, ụ$^{\mathbf{a}}$ … . Sign of equality or identity ↔. Numerals for numbering or

distinguishing variables and evaluating formulae namely, ∅, ⌉, ⌋, 4̶.. n.[5] Mathematical signs for proofs namely, multiplication ⲭ, addition ≻—≺, subtraction ≻— , greater than →, less than ←, greater than or equal to ↳, less than or equal to ↵, division \. Truth constants for signifying true and false formulae or expressions T or ⌉ (true); ⊥, F, ∅ (false), etc.

For non-logical symbols which, includes predicates or relations, functions and constants within the structures of a statement. In our logic, the logician is at liberty to use different non-logical symbols according to the application one has in mind. For this, it is imperative to name the set of all non-logical symbols used in a given application. This is called assignment of signature e.g. Let A be a set of formulae and let B be a formula in a first-order logical system C … A, B and C as used in this signature are non-logical symbols. In western logic there is a traditional approach in which, there is only one language of first-order logic. This practice still persists and some of them may be adopted by an African mathematical logician, example:

- For every integer n ↳ ∅ there is a collection of n-ary, or n-place, predicate symbols, because they represent relations between n elements, they are also called relation symbols. For each arity n we have an infinite supply of them.
  P^n∅, P^n⌉, P^n⌋, P^n 4̶..
- For every integer n ↳ ∅ there are infinitely many n-ary function symbols:
  f^n∅, f^n⌉, f^n⌋, f^n 4̶…

[5] In "An Investigation into the Nature of Mathematical Meaning" *Filosofia Theoretica* 1.1 2011. Pp. 27-28. Chimakonam had first attempted the development of signs of basic numerals from the perspective of African thought system. A better and more concise development however could be found in Chimakonam O. J. "Idea of Africa Numeric System". *Filosofia Theoretica*…2.1. 2013.

As an alternative to the traditional approach, the following may be adopted:

- A predicate symbol or relation symbol with some valence (or arity, number of arguments) greater than or equal to ∅ . These should be denoted by uppercase letters P, R, S …
- Relations of valence ∅ can be identified with propositional variables. For example, P, this can stand for any statement.
- For example, P (ọ) is a predicate variable of valence ˥. One possible interpretation is "ọ is a teacher".
- R (ọụ) is a predicate variable of valence ʇ. Possible interpretations include "ọ is greater than ụ" and ọ is the father of ụ".
- A function symbol, with some valence greater than or equal to∅ These should be denoted by lowercase letters d, e, f, g, …
- Examples: d(ọ) may be interpreted as "the father ọ". In arithmetic, it may stand for "-ọ". In set theory, it may stand for "the power set of ọ". In arithmetic, f (ọ, ụ) may stand for "ọ ⋊ ụ". In set theory, it may stand for "the union ọ and ụ".
- Function symbols of valence ∅ are called constant symbols, and should be denoted by lowercase letters at the beginning of the Igbo alphabet a, b, ch, …, the symbol a may stand for Ezeudo. In arithmetic, it may stand for∅ . In set theory, such a constant may stand for the empty set.

There are also rules that define the terms and formulae of predicate logic. The set of terms is inductively defined by the following rules:

- Variables: any variable is a term
- Functions: any expression (j˥, …, j_n) of n argument (where each argument j_i is a term and g is a function symbol of valence n) is a term. Note therefore that only expressions which can be obtained by finitely many applications of rules and are terms. For example, no expression involving a predicate symbol is a term. On the other hand, the set of

formulae (also called well-formed formulae or wffs is inductively defined by the following rules:

- Predicate symbols: if P is an n-ary predicate symbol and j_1, …, jn terms then P (j_1, …, jn) is a formula.
- Equality: we consider the equality symbol as part of African logic, therefore if and j_1 and j_1 are terms, then $j_1 \leftrightarrow j_1$ is a formula.
- Negation: if ọ is a formula, then ~ ọ is a formula.
- Binary connectives: if ọ and ụ are formulae, then (ọ ⟼ ụ) is a formula; (ọ ∨ ụ) is a formula; (ọ ⟻ ụ) is a formula; and (ọ ⇔ ụ) is a formula, etc.
- Quantifiers: if ọ is a formula and t is a variable, then KW$_{tọ}$ GB$_{tọ}$ and GH$_{tọ}$ are formulae.

Note that only expressions which, can be obtained by finitely many applications of rules – are formulae. The formulae obtained from the first two rules are said to be atomic formulae while that of fourth rule specifically are compound formulae.

Free and bound variables

Variables in any logical formula are either free or bound. A given variable is said to be free if it is not quantified: for example in KWọ P(ụ, ọ), the variable is free while ọ is bound. We may now define inductively the free and bound variable of a formula as follows.

- Atomic formulae: if ị is an atomic formula then ụ is free in ị if and only if ụ occurs in ị. However, there are no bound variables in any atomic formula.
- Negation: ụ is free in ~ ị if and only if ụ is free in ị. ụ is bound in ~ ị if and only if ụ is bound in ị.
- Binary connectives: ụ is free in (ị ⟼ j) if and only if ụ is free in either ị or j. ụ is bound in (ị ⟼ j) if and only ụ is bound in either ị or j. The same rule applies to other binary connectives.
- Quantifiers: ụ is free in KWọị if and only if ụ is free in ị and ụ is a different symbol from ọ. Again, ụ is bound in KWọị if

and only if ụ or ọ or ụ is bound in ị. The same rule applies to GH and GB quantifiers.

However, when a formula in African predicate logic has no free variables it is called first-order or second order sentence such code-named sentences are formulae that have well-defined truth values under an interpretation. In other words, whether a formula such as schol (ụ) is true must depend on what ụ represents. On the other hand, the sentence GHụ schol (ụ) will be either true or false in a given interpretation while just as in KW, that GBụ schol (ụ) is true must also depend on what ụ represents.

3. Semantics

Let us note that for Africans meaning is hidden. Expressions whether in formal or in meta-language mostly do not guide directly to their semantic content. Okonkwo is a tortoise, among the Ibo this does not mean that Okonkwo is an animal but that he is crafty. Likewise most expressions in African natural languages have signatures other than what they seem to contain. We have stated earlier that an assignment of semantic meaning to a logical signature is called interpretation. Now, an interpretation of say a first-order predicate language assigns a denotation to all non-logical constants in that language. In addition, it determines a domain of discourse i.e. subject matter and scope which, specifies the range of the quantifiers. In other words, an interpretation also tells the African logician which variables are free and which are bound by which quantifiers. Normally, under a given interpretation, each term is assigned an object that it represents and each sentence is assigned a truth value but unlike in the western logic, this is not done arbitrarily in African logic. The semantics of African logic is generated from the subject matter or what is called logical custom rather than logical form, the only difficulty is that a non African would have to study the signature of African expressions in order not to be misled by the literally orientation of such expressions. That is to say, it is important to know what an

African means when he utters a logical expression which is most times different from what such expressions literally refer to. On the whole, the truth value assigned to any sentence depends on the semantics of the domain of discourse. We do not for example say “snow falls in Abakaliki during wet season” and assign the value true to it, arbitrarily when we know this to be false in actuality. This is where African logic makes a connection with relevance logic where the claims of the premises must be relevant to the conclusion and the negation of the conclusion in necessarily non-complementary with the premises. So in African logic, an interpretation provides actual semantic meaning to the terms and formulae of the language. The study of the interpretations of customary languages in African logic is called customary semantics, in western logic, it would be formal semantics. Another promising area of African predicate logic is the evaluation of truth values. A formula evaluates to true, true-false or false given an interpretation, and a variable assignment y that associates an element of the domain of discourse with each variable. This is not done arbitrarily and according to the discretion of the African logician but strictly in line with the subject matter or the actual content of the domain of discourse. In other words, formulae and variables are evaluated true, true-false or false in accordance with what they represent in reality. We can map out the following rules for making truth value assignment.

- Variables: each variable ụ with an assignment y evaluates to $y(ụ) \mapsto {}_{M}T(ụ) \vee F(ụ)$
- Functions: given terms $t_1, \ldots, t_n$ that have been evaluated to elements $g_1, \ldots, g_n$ of the domain of discourse, and a n-ary function symbol f, the term $f(t_1, \ldots, t_n)$ evaluates to $(ị(f))(g_1, \ldots, g_n)$.

From here, each formula is assigned a truth value according to the actual value of the subject matter they represent. In fact, in African logic, we do not talk of truth value assignment as though the logicians had the power to do this, what we actually do is to assign subject matter which, each formula or variable would represent. This is where the power and discretion of the African logician ends, the values for such formulae or variables naturally reveal themselves to the logician. Hence, the values to be assigned to any given formula and variable in African logic are determined by the subject-matter of the domain of discourse. The inductive definition used to make this truth value assignment we shall here call the R-schema. In western logic, it would be the Alfred Tarksi's T-schema[6] due to the fact that truth values in western logic are arbitrarily assigned following the discretion of the western logician.

R-schema in African logic can be stated thus:

$F \mapsto_M S \leftrightarrow T$

Where F symbolizes functions, S for subject matter and T for truth value, the R-schema states that every function i.e. formulae or variable has a subject matter assigned to it or it represents and the truth value of such a formula or variable depends entirely on the actual content of the subject matter it represents. R-schema therefore simply means relevance-schema because African logicians insist that the evaluation of their logical formulae be relevant to the subject matter. The inductive definition for R-schema is as follows:

- Atomic formula (ˀ1): A formula P (t_1, …, t_n) is assigned the value true, true-false or false depending on whether (v_1, … v_n) $\in$ I(P), where v_1, …, v_n are the evaluation of the terms t_1, …t_n and ị (P) is the interpretation of P, which, by assumption is a subject of D^n (infinite domain of discourse). Note of course

[6] Tarski, Alfred. "The Semantic Conception of Truth and the Foundations of Semantics". *Philosophy and Phenomenological Research*. 4 (1944): 341 – 376. Print. See also his work "The Concept of Truth in the Languages of the Deductive Sciences". *Studia Philosophica*. (1933 and 1935): 261 – 405. Print.

that ị (P) and D^n are not arbitrary signatures unless stated otherwise. African logicians may sometimes choose to assign signatures in an interpretation from a possible rather than the para-contingent world. However, when this is done, it is stated in the interpretation. The evaluation of formulae in such domain of discourse generally becomes modal and inferential. Similarly, when signatures are assigned arbitrarily as most times is the case in western logic, the evaluation of formulae becomes a formal exercise. In African logic, we describe such as restrictive logic (RL) in the sense that evaluation has been restricted to logical form and logical custom (relevance) thrown over-board. This type of logic is done to exercise the mind rather than to obtain good reasoning.

- Atomic formulae (˩): a formula t˥ ↔ t˩ is assigned true if t˥ and t˩ evaluate to the same object of the domain of discourse.

- Logical connectives: a formula in the form of ~ ọ, ọ |⟶ $_M$ ụ etc., is evaluated according to the truth table method (TTM), truth funnel method (TFM) or short proof method (SPM)[7] for the connective in question, as in propositional logic earlier discussed.
- Existential quantifiers (one and some): the quantifier GHeụ (e) and GBeụ(e) are true if and only if there is a way to choose a value for e such that ọ (e) is satisfied. This entails that ọ is a subject of ụ, thus if e is satisfied in ụ it would be satisfied in ọ as well given the same context. But the hub of this decision starts from the subject matter ụ represents.

[7] In Chimakonam, Okeke, Jonathan. *Introducing African logic and Numeric System: Formalist and Axiomatic Approach*. (Forthcoming), extensive developments and applications of some of these proof methods were carried out under the African propositional logic.

- Universal quantifier: the formula KWeụ (e) is true if every possible choice of a value for e causes ọ (e) to be true. For this to hold, ọ must be a subset of ụ and the interpretation given ụ must be actual. Based on this, if ụ actually satisfies e then every possible subset of ụ would satisfy e given the same context.

4. Contexts, Worlds and Quantifiers

There are three worlds in African universe namely: ụwa (material), elu-igwe (anti-material) and ala-mmụọ (non-material) which translate to the three contexts para-contingent, necessary and possible symbolized respectively as M, A, N (universals) and m, a, n (particulars)[8]. In African logic these are variously expressed as :

a. For all things para-contingent…KW(M)
b. For all things necessary…KW(A)
ch. For all things possible…KW(N)
d. There are some things para-contingent…GH(m)
e. There is a thing para-contingent…GB(m)
f. There are some things necessary…GH(a)
g. There is a thing necessary…GB(a)
gb. There are some things possible…GH(n)
gh. There is a thing possible…GB(n)

In the above, Igbo twin upper case letters KW, GH, GB are used as universal and existential quantifiers (some and one) respectively. Hence a propositional function as f perm $\mapsto_{M}$ g would be read as f wedge-implies g in all things para-contingent. The wider implication here is that whenever f is stated g may and may not follow since para-contingence depicts a context that is both contingent and necessary depending on existential circumstances. The same goes for the existential version where the truth-value

[8] For initial extensive treatment of the M-A-N contexts in African logic see Chimakonam O. J. *Introducing African Science: Systematic and Philosophical Approach*. Bloomington Indiana: Authorhouse, 2012. Pp. 25-34

also depends on both logical custom and logical form. But for all things necessary and its existential version, the truth-value which is definitely true or false depends on logical form. However, for all things possible and its existential version, the truth-value depends on logical custom rather than on logical form and is said to be complemented. This is because the possible world that might have been is also a world that permanently is and it is different from the para-contingent world that may and may not be, and the necessary world that simply is. The further difference between the necessary world that simply is, and the possible world that permanently is, is that the former is a partial realization of value whereas the latter is a full or complete realization of value. Although the possible world is also a world that might have been if fragmented, it is nonetheless permanently is. This is called truth-value glut where logical functions or constants complement themselves (see the section on complementary mode)

5. Soundness, validity, satisfiability and wedged-consequence

If a sentence ọ evaluates to true under a given interpretation H, one says that H satisfies ọ; this is symbolized H ⊨ọ. A sentence is satisfiable if there is some interpretation under which, it is true through a relevant context, hence the formula is logically sound or simply sound; if it is inconsistent in some interpretation then it is valid. These formulae play role similar to tautologies in propositional logic. Finally, a formula ọ is a wedged-consequence of a formula ụ if every interpretation that makes ụ true also makes ọ true through a relevant context. In this case one says that ọ is wedge-implied by ụ. Elsewhere[9], I have undertaken the task of this section in clearer detail.

[9] Chimakonam, Okeke, Jonathan. *Introducing African logic and Numeric System: Formalist and Axiomatic Approach*. (Forthcoming)